Journal of Applied Mechanics and Technical Physics, Vol. 40, No. 1, 1999

MODELING OF PLANE-WAVE PROPAGATION IN AN ANISOTROPIC ELASTIC MEDIUM

S. A. Anisimov and I. O. Bogul'skii[1]

UDC 539.3

We propose an algorithm that reduces the process of numerical solution to successive calculation of elementary one-dimensional problems of the type of a system of acoustic equations.

Introduction. To model the propagation of plane waves in an anisotropic inhomogeneous elastic body, it is necessary to solve a mixed problem for a one-dimensional system of coupled hyperbolic equations. The algorithm of numerical solution of these problems is well known [1, 2] and based on reduction of the system to the canonical form. In the case of a system of large dimension, this procedure encounters considerable technical difficulties. In the present paper, we propose an algorithm the essence of which is to reduce the process of numerical solution to successive calculation of elementary one-dimensional problems of the type of a system of acoustic equations.

Plane Waves in an Anisotropic Elastic Layer. We shall consider the process of wave propagation along the z axis in a laminated inhomogeneous elastic layer $0 \leqslant z \leqslant L$ which is a set of K elastic layers of constant thickness H_i ($i = 1,\ldots,K$) which are infinite in the x and y directions (Fig. 1). These layers are assumed to be transversely isotropic. For each of the layers, for a transversely isotropic medium (hexagonal system), in the coordinate system (x', y', z') conformed with the crystallographic axis of the material the Hooke's law can be written in the form [3]

$$\varepsilon_x = \frac{1}{E_1}(\sigma_x - \nu_1\sigma_y) - \frac{\nu_2}{E_2}\sigma_z, \quad \tau_{xy} = 2\mu_1\varepsilon_{xy}, \quad \varepsilon_y = \frac{1}{E_1}(\sigma_y - \nu_1\sigma_x) - \frac{\nu_2}{E_2}\sigma_z,$$
$$\tau_{xz} = 2\mu_2\varepsilon_{xz}, \qquad \varepsilon_x = -\frac{\nu_2}{E_2}(\sigma_x + \sigma_y) + \frac{1}{E_2}\sigma_z, \qquad \tau_{yz} = 2\mu_2\varepsilon_{yz}. \tag{1}$$

The Young's moduli E_1 and E_2, the Poisson ratios ν_1 and ν_2, and the shear moduli μ_1 and μ_2 which enter (1) are connected by the supplementary relation $E_1 = 2\mu_1(1 + \nu_1)$.

Thus, the transversely isotropic medium in each layer is characterized by five independent moduli of elasticity, density, and inclination of the crystallographic axes with respect to the axes of the Cartesian coordinate system (x, y, z).

We now formulate some assumptions simplifying the problem. Let the boundary conditions at the surfaces perpendicular to the z axis ($z = 0$ and $z = L$) and the initial stresses and mass velocity of the particles be independent of the coordinates x and y for $t = 0$. Moreover, we assume that the (z', x') plane coincides with the (z, x) plane so that the inclination of the crystallographic axis is uniquely determined by the angle of slope φ of the z' axis to the z axis, and the velocity vector always lies in the (z, x) plane. Thus, we consider the one-dimensional problem, i.e., the desired functions depend only on the spatial variable z and the time t.

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. [1]Institute of Computational Modeling, Siberian Division, Russian Academy of Sciences, Krasnoyarsk 660036. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 40, No. 1, pp. 199–206, January–February, 1999. Original article submitted May 26, 1997.

 0021-8944/99/4001-0172 $22.00

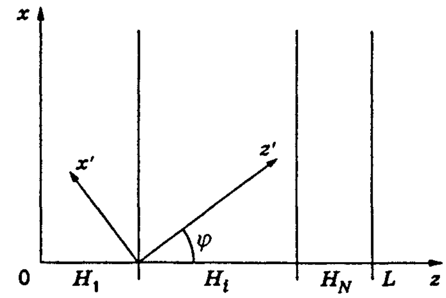

Fig. 1

As desired functions, we consider two components of the mass velocity vector u and v along the z and x directions, respectively, and the normal σ_z and shear τ_{zx} components of the stress tensor in the coordinate system (z, x). They satisfy the equations of motion

$$\rho \frac{\partial u}{\partial t} = \frac{\partial \sigma_z}{\partial z}, \qquad \rho \frac{\partial v}{\partial t} = \frac{\partial \tau_{zx}}{\partial z} \tag{2}$$

and the Hooke's law; after differentiation with respect to the time this law takes the form

$$\frac{\partial \sigma_z}{\partial t} = A \frac{\partial u}{\partial z} + B \frac{\partial v}{\partial z}, \qquad \frac{\partial \tau_{zx}}{\partial t} = B \frac{\partial u}{\partial z} + C \frac{\partial v}{\partial z}. \tag{3}$$

Here

$$A = a \cos^4 \varphi + 2b \sin^2 \varphi \cos^2 \varphi + c \sin^4 \varphi + \mu_2 \sin^2 2\varphi, \quad C = \frac{1}{4}(a - 2b + c) \sin^2 2\varphi + \mu_2 \cos^2 2\varphi,$$

$$B = -\frac{1}{2}(a \cos^2 \varphi - b \cos 2\varphi - c \sin^2 \varphi) \sin 2\varphi + \mu_2 \cos 2\varphi \sin 2\varphi, \tag{4}$$

$$a = \frac{(1 - \nu_1)E_2^2}{E_2(1 - \nu_1) - 2E_1\nu_2^2}, \qquad b = \frac{\nu_2 E_1 E_2}{E_2(1 - \nu_1) - 2E_1\nu_2^2}, \qquad c = \frac{E_1(E_2 - \nu_2^2 E_1)}{(1 + \nu_1)E_2(1 - \nu_1) - 2E_1\nu_2^2}.$$

Bearing in mind (4), one can readily verify that the coefficient matrix at the derivatives with respect to z on the right-hand side of (3) is positive definite. For each layer, we have a one-dimensional system of four equations (2) and (3) of the hyperbolic type [1] to determine four desired functions u, v, σ_z, and τ_{zx}:

$$\frac{\partial}{\partial t} \begin{pmatrix} u \\ v \\ \sigma_z \\ \tau_{zx} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1/\rho & 0 \\ 0 & 0 & 0 & 1/\rho \\ A & B & 0 & 0 \\ B & C & 0 & 0 \end{pmatrix} \frac{\partial}{\partial z} \begin{pmatrix} u \\ v \\ \sigma_z \\ \tau_{zx} \end{pmatrix}. \tag{5}$$

The propagation velocity of disturbances in the material described is determined by the slope of the characteristics of the system $dz/dt = \pm c_+$ and $dz/dt = \pm c_-$, which are calculated as eigenvalues of the coefficient matrix on the right-hand side of (5):

$$c_+^2 = \frac{A + C + \sqrt{(A - C)^2 + 4B^2}}{2\rho}, \qquad c_-^2 = \frac{A + C - \sqrt{(A - C)^2 + 4B^2}}{2\rho}.$$

We note that if $A > C$, we have $c_- \leqslant \sqrt{c}/\rho < \sqrt{A}/\rho \leqslant c_+$.

We consider the following problem for systems (2) and (3). At the initial moment of time, all the desired functions are specified:

$$u(0, z) = u^0(z), \quad v(0, z) = v^0(z), \quad \sigma_z(0, z) = \sigma_z^0(z), \quad \tau_{zx}(0, z) = \tau_{zx}^0(z); \tag{6}$$

at the surfaces $z = 0$ and $z = L$, we confine ourselves to the boundary conditions of the form

$$(\psi_1^{\pm} u + \chi_1^{\pm} \sigma_z)\Big|_{z=0,L} = f_1^{\pm}, \qquad (\psi_2^{\pm} v + \chi_2^{\pm} \tau_{zx})\Big|_{z=0,L} = f_2^{\pm}, \tag{7}$$

where $\psi_i^{\pm}$, $\chi_i^{\pm}$, and $f_i^{\pm}$ ($i = 1, 2$) are the specified time-dependent functions. The adjacent layers are perfectly conjugated, i.e., the stress and velocity vectors are continuous at their boundaries.

Numerical Solution Based on Vector Decomposition. If the layers considered have different mechanical properties, the formulated problem can be solved only numerically. If the material is isotropic, i.e., $E_1 = E_2$, $\nu_1 = \nu_2$, and $\mu_1 = \mu_2$, or the angle of inclination φ of the z' axis to the z axis equals either zero or $\pi/2$, we have $B = 0$, and the problem (5)-(7) is completely decomposed into two independent problems of the form

$$\rho \frac{\partial W}{\partial t} = \frac{\partial P}{\partial z}, \qquad \frac{\partial P}{\partial t} = D \frac{\partial W}{\partial z}, \tag{8}$$

$$W(0, z) = W^0(z), \quad P(0, z) = P^0(z), \quad (\psi^{\pm} W + \chi^{\pm} P)\big|_{z=0,L} = f^{\pm},$$

where by W and P we mean u and σ_z with $D = A$ in the first problem and v and τ_{zx} with $D = C$ in the second problem.

Using a special discretization of the calculation domain which agrees with the disturbance velocity, one can obtain exact solutions of both problems. In the case $B \neq 0$, the method of solution of the complete problem is well known [1, 2] and consists of the diagonalization of the matrix in (5).

We write system (5) in the form

$$\rho \frac{\partial \mathbf{W}}{\partial t} = \frac{\partial \mathbf{P}}{\partial z}, \qquad \frac{\partial \mathbf{P}}{\partial t} = D \frac{\partial \mathbf{W}}{\partial z}, \tag{9}$$

where

$$\mathbf{W} = \begin{pmatrix} u \\ v \end{pmatrix}, \qquad \mathbf{P} = \begin{pmatrix} \sigma_z \\ \tau_{zx} \end{pmatrix}, \qquad D = \begin{pmatrix} A & B \\ B & C \end{pmatrix}.$$

We divide the calculation domain lying in the (z, t) plane into the elementary rectangles $\Omega = \{z_j \leqslant z \leqslant z_{j+1}, t_k \leqslant t \leqslant t_{k+1}\}$ by straight lines parallel to the z and t axes and introduce a local coordinate system (ξ, η) in each of them:

$$\xi = \frac{2}{h}\Big[z - \frac{1}{2}(z_j + z_{j+1})\Big], \quad \xi = \frac{2}{\tau}\Big[t - \frac{1}{2}(t_k + t_{k+1})\Big], \quad h = z_{j+1} - z_j, \quad \tau = t_{k+1} - t_k.$$

As the approximate solution in Ω, we use the polynomials linear in ξ and η

$$\mathbf{W} = \mathbf{W}_0 + \mathbf{W}_1 \eta, \quad \mathbf{P} = \mathbf{P}_0 + \mathbf{P}_1 \eta, \quad \mathbf{W}' = \mathbf{W}_0' + \mathbf{W}_1' \xi, \quad \mathbf{P}' = \mathbf{P}_0' + \mathbf{P}_1' \xi \tag{10}$$

which satisfy the system

$$\rho \frac{\partial \mathbf{W}}{\partial t} = \frac{\partial \mathbf{P}'}{\partial z}, \qquad \frac{\partial \mathbf{P}}{\partial t} = D \frac{\partial \mathbf{W}'}{\partial z}. \tag{11}$$

To calculate $\mathbf{W}$ and $\mathbf{P}$ at the upper time layer the following formulas are derived from (10) and (11):

$$\mathbf{W}^{j+1/2} = \mathbf{W}_{j+1/2} + \frac{\tau}{\rho h}(\mathbf{P}_{j+1} - \mathbf{P}_j), \qquad \mathbf{P}^{j+1/2} = \mathbf{P}_{j+1/2} + \frac{\tau}{\rho h} D(\mathbf{W}_{j+1} - \mathbf{W}_j), \tag{12}$$

where

$$\mathbf{W}_{j+1/2} = \mathbf{W}\Big|_{\eta=-1}, \qquad \mathbf{W}^{j+1/2} = \mathbf{W}\Big|_{\eta=1}, \qquad \mathbf{W}_{j+1} = \mathbf{W}'\Big|_{\xi=1}, \qquad \mathbf{W}_j = \mathbf{W}'\Big|_{\xi=-1},$$

$$\mathbf{P}_{j+1/2} = \mathbf{P}\Big|_{\eta=-1}, \qquad \mathbf{P}^{j+1/2} = \mathbf{P}\Big|_{\eta=1}, \qquad \mathbf{P}_{j+1} = \mathbf{P}'\Big|_{\xi=1}, \qquad \mathbf{P}_j = \mathbf{P}'\Big|_{\xi=-1}.$$

To find the quantities with integer indices, supplementary equations can be constructed based on the following energy identity, which is valid for all the polynomials in (10) satisfying (11):

$$\iint_\Omega \rho \mathbf{W}_0 \frac{\partial \mathbf{W}}{\partial t}\, d\Omega + \iint_\Omega \mathbf{P}_0 \frac{\partial \mathbf{W}'}{\partial z}\, d\Omega + \iint_\Omega Q\, d\Omega = \iint_\Omega \frac{\partial \mathbf{W}'\mathbf{P}'}{\partial z}\, d\Omega,$$

where Q is the power of the artificial energy dissipation; with allowance for (10) and (11), this power can be written in the form

$$Q = \left(\mathbf{W}_0' - \mathbf{W}_{j+1/2} - \frac{\tau}{2\rho}\frac{\partial \mathbf{P}'}{\partial z}\right)\frac{\partial \mathbf{P}'}{\partial z} + \left(\mathbf{P}_0' - \mathbf{P}_{j+1/2} - \frac{\tau}{2} D \frac{\partial \mathbf{W}'}{\partial z}\right)\frac{\partial \mathbf{W}'}{\partial z}. \tag{13}$$

We represent the matrix D in the form

$$D = T\tilde{D}T^{-1}, \qquad \tilde{D} = \begin{pmatrix} æ_1 & 0 \\ 0 & æ_2 \end{pmatrix}, \qquad æ_1 = \rho c_+^2, \qquad æ_2 = \rho c_-^2,$$

where $\tilde{D}$ is the diagonal matrix and T is the matrix composed of the normalized eigenvectors of the matrix D which correspond to the eigenvalues $æ_1$ and $æ_2$:

$$T = \begin{pmatrix} -\theta & 1 \\ 1 & \theta \end{pmatrix}, \qquad T^{-1} = -\frac{1}{1+\theta^2}\begin{pmatrix} \theta & -1 \\ -1 & -\theta \end{pmatrix}, \qquad \theta = \frac{B}{æ_2 - æ_1}.$$

We formulate the supplementary equations in the form

$$\mathbf{W}_0' - \mathbf{W}_{j+1/2} - \frac{1}{2}T\begin{pmatrix} (\tau+\omega_1)/\rho & 0 \\ 0 & (\tau+\omega_2)/\rho \end{pmatrix} T^{-1}\frac{\partial \mathbf{P}'}{\partial z} = 0,$$

$$\mathbf{P}_0' - \mathbf{P}_{j+1/2} - \frac{1}{2}T\begin{pmatrix} (\tau+\gamma_1)æ_1 & 0 \\ 0 & (\tau+\gamma_2)æ_2 \end{pmatrix} T^{-1}\frac{\partial \mathbf{W}'}{\partial z} = 0.$$

Thereafter, the system of equations for determination of the integer quantities can be written in the form

$$\begin{pmatrix} \mathbf{W}_{j+1} + \mathbf{W}_j \\ \mathbf{P}_{j+1} + P_j \end{pmatrix} - M\begin{pmatrix} \mathbf{W}_{j+1} - \mathbf{W}_j \\ \mathbf{P}_{j+1} - \mathbf{P}_j \end{pmatrix} = 2\begin{pmatrix} \mathbf{W}_{j+1/2} \\ \mathbf{P}_{j+1/2} \end{pmatrix},$$

where M is a 4×4 matrix whose elements depend on the dissipation constants $\omega_1, \omega_2, \gamma_1$, and γ_2. As is shown in [4], the nonnegativity condition for the dissipation constants ensures the stability of the approximate solution and the convergence to the exact solution. Moreover, the formulas for calculation of $\mathbf{W}_j$ and $\mathbf{P}_j$ will be explicit provided the eigenvalues of M are equal to 1 or -1.

To calculate the quantities with integer indices, we use the explicit formulas

$$\left(\theta d_- u - d_- v + \frac{\theta}{d_-}\sigma_z - \frac{1}{d_-}\tau_{zx}\right)_j = \left(\theta d_- u - d_- v + \frac{\theta}{d_-}\sigma_z - \frac{1}{d_-}\tau_{zx}\right)_{j+1/2},$$

$$\left(-d_+ u - \theta d_+ v - \frac{1}{d_+}\sigma_z - \frac{\theta}{d_+}\tau_{zx}\right)_j = \left(-d_+ u - \theta d_+ v - \frac{1}{d_+}\sigma_z - \frac{\theta}{d_+}\tau_{zx}\right)_{j+1/2},$$

$$\left(-\theta d_- u + d_- v + \frac{\theta}{d_-}\sigma_z - \frac{1}{d_-}\tau_{zx}\right)_{j+1} = \left(-\theta d_- u + d_- v + \frac{\theta}{d_-}\sigma_z - \frac{1}{d_-}\tau_{zx}\right)_{j+1/2},$$

$$\left(d_+ u + \theta d_+ v - \frac{1}{d_+}\sigma_z - \frac{\theta}{d_+}\tau_{zx}\right)_{j+1} = \left(d_+ u + \theta d_+ v - \frac{1}{d_+}\sigma_z - \frac{\theta}{d_+}\tau_{zx}\right)_{j+1/2}$$

($d_- = \sqrt{\rho c_-}$ and $d_+ = \sqrt{\rho c_+}$), the continuity conditions for the quantities u_j, v_j, $(\sigma_z)_j$, and $(\tau_{zx})_j$ at the common boundaries between two adjacent cells, and the boundary conditions (7). The scheme is stable if the following restriction imposed on the time step τ holds true:

$$c_+\tau \leqslant h. \tag{14}$$

It should be noted that the scheme possesses a positive artificial dissipation proportional to the quantity $(c_+/c_- - 1)$ even in the case of equality in (14), and the discontinuity will be smeared at the wavefront

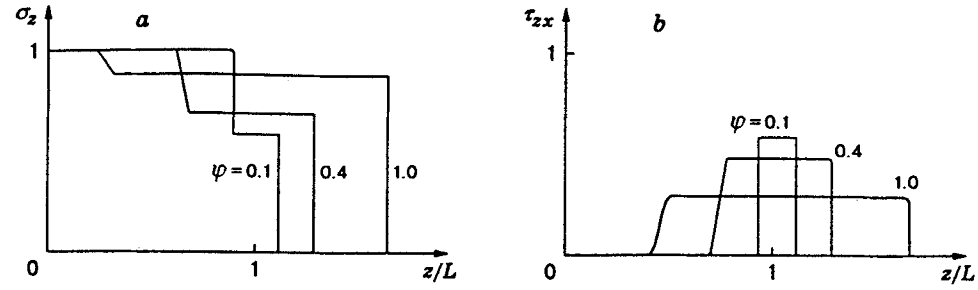

Fig. 2

moving with velocity c_- in all cases where $c_- < c_+$.

Figure 2 shows the results of test calculations of the problem formulated for a single layer when $E_1 = E_2$, $\nu_1 = \nu_2$, $\mu_1 = 0$, and $\nu_2 E_2/(1+\nu_2)(1-2\nu_2) = \mu_2$. In this case, we have $A = \mu_2(1+\sin^2 2\varphi)$, $C = \mu_2 \cos^2 2\varphi$, and $B = \mu_2 \cos 2\varphi \sin 2\varphi$. The diagrams of the normal σ_z and shear τ_{zx} stresses are plotted for $\varphi = 0.1$, 0.4, and 1. The results obtained at the 200th time step are given in Fig. 2. The zero initial conditions are assumed and the normal impact excitation $\sigma_z = 1$ is applied at the surface $z = 0$. It is noteworthy that, in contrast to the case of an isotropic material, both graphs have two jumps even when a normal impact occurs.

Iterative Procedure for Solution of the Problem. The above algorithm consists of reducing the initial system of equations to a canonical form for each of the layers. In the case of a high-order matrix, this procedure encounters significant technical difficulties. For example, rejection of the assumption of plane-parallel motion would result in a system of six coupled equations. The problem becomes much more complicated in the two-dimensional case.

In [4], the authors proposed an iterative procedure to solve two-dimensional dynamic problems of the theory of elasticity, which is based on the two-stage solution of one-dimensional problems into which the two-dimensional problem is decomposed. The essence of the procedure is to take into account the "interfering" terms of the equations as "correcting" terms only at the second stage. We use this computational technique to solve system (5).

The procedure of numerical solution of system (9) remains the same up to the formulation of supplementary equations. Dividing the calculation domain into the elementary rectangles Ω and taking the linear polynomials (10) as an approximate solution in Ω, we obtain formulas for calculation of the approximate solution at the upper time layer:

$$u^{j+1/2} = u_{j+1/2} + \frac{\tau}{\rho h}[(\sigma_z)_{j+1} - (\sigma_z)_j], \qquad v^{j+1/2} = v_{j+1/2} + \frac{\tau}{\rho h}[(\tau_{zx})_{j+1} - (\tau_{zx})_j],$$
$$\sigma_z^{j+1/2} = (\sigma_z)_{j+1/2} + \frac{\tau}{h}A(u_{j+1} - u_j) + \frac{\tau}{h}B(v_{j+1} - v_j), \tag{15}$$
$$\tau_{zx}^{j+1/2} = (\tau_{zx})_{j+1/2} + \frac{\tau}{h}B(u_{j+1} - u_j) + \frac{\tau}{h}C(v_{j+1} - v_j).$$

We expand the artificial-dissipation power Q (13) in the form

$$Q = \Big(u_0' - u_{j+1/2} - \frac{\tau}{2\rho}\frac{\partial \sigma_z'}{\partial z}\Big)\frac{\partial \sigma_z'}{\partial z} + \Big((\sigma_z)_0' - (\sigma_z)_{j+1/2} - \frac{\tau}{2}A\frac{\partial u'}{\partial z} - \frac{\tau}{2}B\frac{\partial v'}{\partial z}\Big)\frac{\partial u'}{\partial z}$$
$$+ \Big(v_0' - v_{j+1/2} - \frac{\tau}{2\rho}\frac{\partial \tau_{zx}'}{\partial z}\Big)\frac{\partial \tau_{zx}'}{\partial z} + \Big((\tau_{zx})_0' - (\tau_{zx})_{j+1/2} - \frac{\tau}{2}C\frac{\partial v'}{\partial z} - \frac{\tau}{2}B\frac{\partial u'}{\partial z}\Big)\frac{\partial v'}{\partial z}.$$

The supplementary equations can be formulated in the form

$$u_0' - u_{j+1/2} - \frac{\tau}{2\rho}\frac{\partial \sigma_z'}{\partial z} = \frac{\gamma}{2\rho}\frac{\partial \sigma_z'}{\partial z}, \quad (\sigma_z)_0' - (\sigma_z)_{j+1/2} - \frac{\tau}{2}A\frac{\partial u'}{\partial z} = \frac{\gamma}{2}A\frac{\partial u'}{\partial z} + \alpha\tau B\Big(\frac{\partial u'}{\partial z}\Big)^*; \tag{16}$$

$$v_0' - v_{j+1/2} - \frac{\tau}{2\rho}\frac{\partial \tau_{zx}'}{\partial z} = \frac{\omega}{2\rho}\frac{\partial \tau_{zx}'}{\partial z}, \quad (\tau_{zx})_0' - (\tau_{zx})_{j+1/2} - \frac{\tau}{2}C\frac{\partial v'}{\partial z} = \frac{\omega}{2}C\frac{\partial v'}{\partial z} + \beta\tau B\Big(\frac{\partial u'}{\partial z}\Big)^*, \tag{17}$$

where α, β, γ, and ω are the dissipation constants. The solutions of the one-dimensional problems (16) and (17) are constructed in two stages. At the first stage, we set α and β to be zero, solve two independent problems of the form (8) (for an isotropic medium), and denote the resulting derivatives by $(\partial u'/\partial z)^*$ and $(\partial v'/\partial z)^*$. At the second stage, we solve two independent problems of the form (8) by setting α and β equal to 1/2 (see [4]). Here the artificial-dissipation power has the form

$$Q = \frac{\gamma}{2\rho}\Big(\frac{\partial \sigma_z'}{\partial z}\Big)^2 + \frac{\omega}{2\rho}\Big(\frac{\partial \tau_{zx}'}{\partial z}\Big)^2 + \frac{\gamma}{2}A\Big(\frac{\partial u'}{\partial z}\Big)^2$$
$$-\tau B\Big[\frac{\partial u'}{\partial z}\frac{\partial v'}{\partial z} - \alpha\frac{\partial u'}{\partial z}\Big(\frac{\partial v'}{\partial z}\Big)^* - \beta\Big(\frac{\partial u'}{\partial z}\Big)^*\frac{\partial v'}{\partial z}\Big] + \frac{\omega}{2}C\Big(\frac{\partial v'}{\partial z}\Big)^2.$$

The solutions of the one-dimensional problems (16) and (17) are calculated explicitly if $\gamma = h/\sqrt{A/\rho} - \tau$ and $\omega = h/\sqrt{C/\rho} - \tau$.

One can show that the approximate solution converges to an exact solution if the inequalities $\gamma \geqslant 0$ and $\omega \geqslant 0$, from which the restriction imposed on the time step follows,

$$\tau \leqslant h/\sqrt{\max(A, C)/\rho}, \tag{18}$$

are satisfied.

Thus, the algorithm of solution of the complete problem excludes the matrix diagonalization procedure and reduces to the solution of a necessary set of hyperbolic systems of two equations (systems of acoustic equations).

It should be noted that, as in the above algorithm, the use of a time step which is the maximum admissible according to (18) in the scheme does not allow one to obtain an exact solution of the problem, since the one-dimensional problems (16) and (17), which describe the propagation of disturbances with different velocities, are solved independently but with the use of the same grid, which implies that Q does not vanish and is proportional to the factor $\sqrt{\max(A, C)/\min(A, C)} - 1$.

The diagrams of the stresses σ_z and τ_{zx} and of the velocities, which coincide with those in Fig. 2, were obtained for the 200th time step on the basis of test calculations according to the scheme (15)–(17).

As another example, we consider the problem of wave passage through a multilayered elastic obstacle. It is well known [5] that lamination can significantly improve the screening properties of a structure. We consider a laminated material representing a stack of twenty plates of thickness z perpendicular to the H axis. These plates are cut from the same, transversely elastic material so that the crystallographic axis z' in each layer coincides with the z axis ($\varphi = 0$) or is perpendicular to it ($\varphi = \pi/2$). The layers alternate, and the velocity of the longitudinal waves in each even layer is twice that in the odd layer.

We assume that a plane monochromatic longitudinal wave is incident on the stack at a right angle from the half-space $z < 0$. For $z = 0$, the boundary conditions in this case have the form

$$(\sigma_z - \rho c_p u)\Big|_{z=0} = \sin(2\pi k c_p t/H), \qquad \tau_{zx}\Big|_{z=0} = 0,$$

where k is the number of waves along the layer of thickness H. At the right end, we formulate the nonreflecting conditions [4]

$$(\sigma_z + \rho c_p u)\Big|_{z=L} = 0, \qquad (\tau_{zx} + \rho c_s v)\Big|_{z=L} = 0$$

(c_p and c_s are the velocities of the longitudinal and transverse waves, respectively).

If the calculations are sufficiently prolonged, a certain quasistationary regime is reached. In [5], the following problem is formulated: from a given set of materials, it is required to design a structure that ensures maximum damping of the wave energy flux propagated into the half-space $z > L$. We consider a simpler problem: to determine the number k (i.e., the frequency of the incident wave) for which the wave energy flux

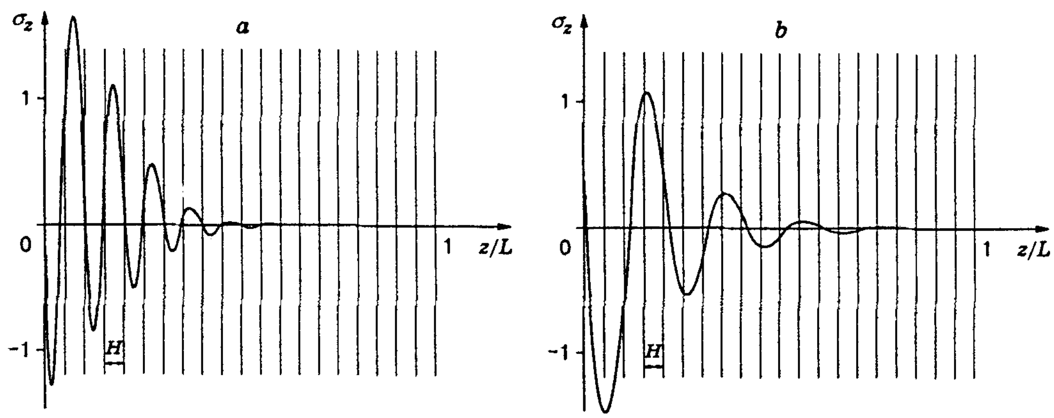

Fig. 3

characterized by the squared ratio between the amplitudes of the passing and incident waves is minimum. For the laminated structure considered, the numerical experiment gives $k \approx 0.7$. Figure 3a shows the diagram of σ_z for the 2000th time step (for this time the wave runs along the stack about 6.5 times). We now consider the same problem for a stack composed of the same number of layers for the case where the odd layers have zero inclination of the z' axis with respect to the z axis, while the even layers have an inclination equal to $\pi/6$. We use the following values of the layer parameters: $\nu_1 E_1/(1+\nu_1)(1-2\nu_1) = 2\mu_1$ and $\mu_2 = 2\mu_1$; then $A = 4\mu_1$, $C = 2\mu_1$, and $B = 0$ for odd layers and $A = 19\mu_1/4$, $C = 5\mu_1/4$, and $B = \sqrt{3}\mu_1/4$ for even layers. We assume that a monochromatic wave of the same intensity as in the previous problem falls from the half-space $z < 0$ at an angle $\pi/4$ to the stack. The diagram of the normal stress σ_z at the 1700th time step is shown in Fig. 3b. The calculations give $k \approx 0.35$.

This work was supported by the Russian Foundation for Fundamental Research (Grant No. 97-01-00434).

REFERENCES

1. S. K. Godunov, A. V. Zabrodin, M. Ya. Ivanov, et al., *Numerical Solution of Multidimensional Problems of Gas Dynamics* [in Russian], Nauka, Moscow (1976).
2. S. A. Anisimov and I. O. Bogul'skii, "Algorithm of independent approximation of nondifferential terms in numerical solution of boundary-value problems for systems of hyperbolic equations," in: *Dynamics of Continuous Media* (collected scientific papers) [in Russian], Novosibirsk, **109** (1994), pp. 34–48.
3. T. D. Shermergor, *Theory of Elasticity of Microhomogeneous Media* [in Russian], Nauka, Moscow (1977).
4. S. A. Anisimov and I. O. Bogul'skii, *Numerical Solution of the Dynamic Problems of Elastic Bodies* [in Russian], Izd. Novosibirsk Univ., Novosibirsk (1995).
5. M. A. Kanibolotskii and Yu. S. Urzhumtsev, *Optimal Design of Laminated Structures* [in Russian], Nauka, Novosibirsk (1989).